On Cutting Cakes and Crossing Curves

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Lund University

EC 2026

Joint work with Gilbert Maystre and Alexandros Hollender

Cake Cutting

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- n agents

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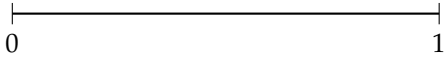

Cake Cutting

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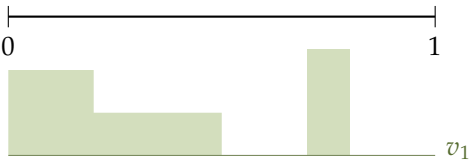

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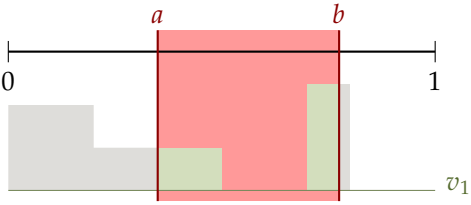

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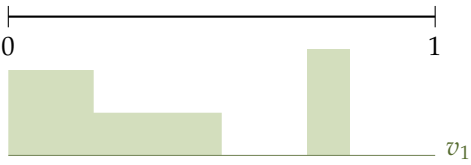

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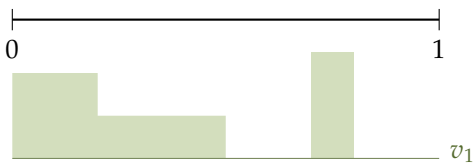

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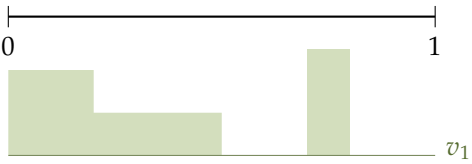

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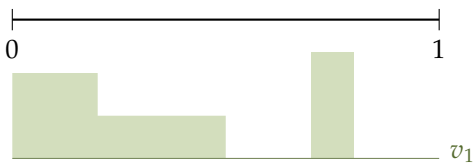

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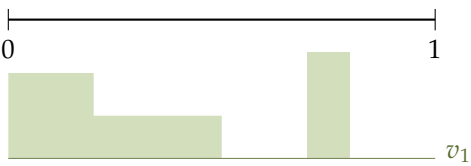

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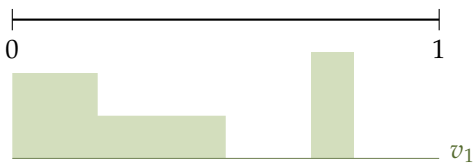

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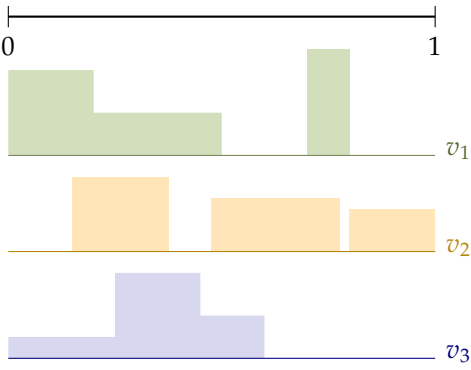

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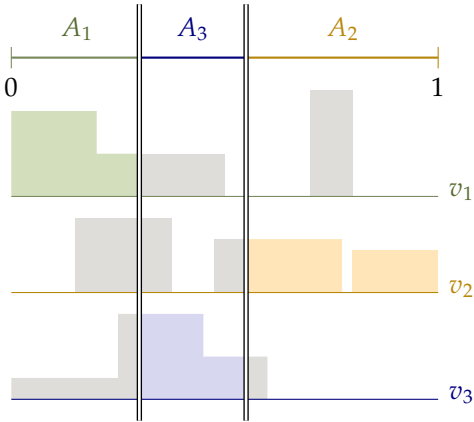

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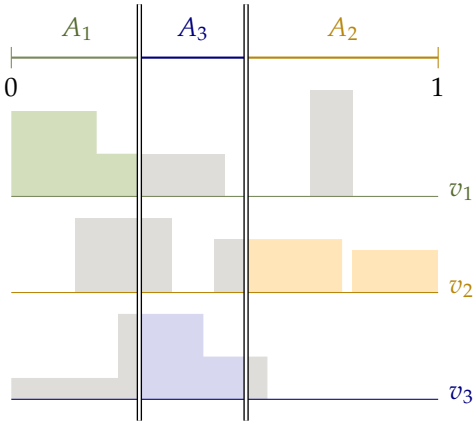

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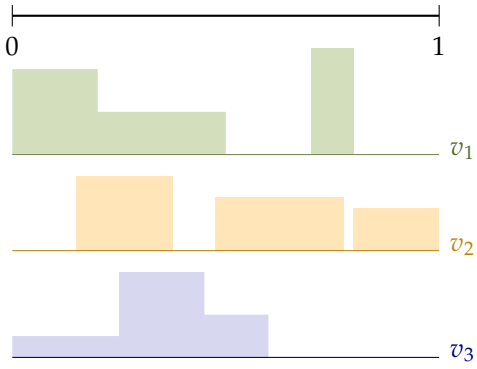
$\Rightarrow$ exists always (by fixed-point theorem) [Str80, Woo80]

Cake Cutting: Model of Computation

Query Model:



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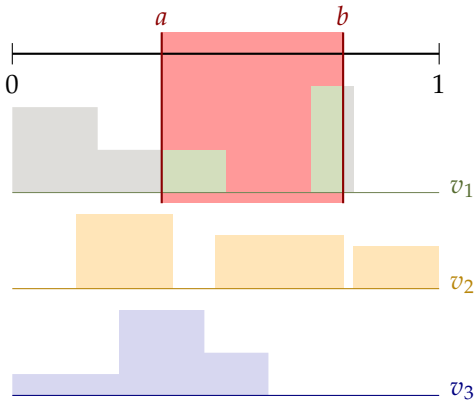

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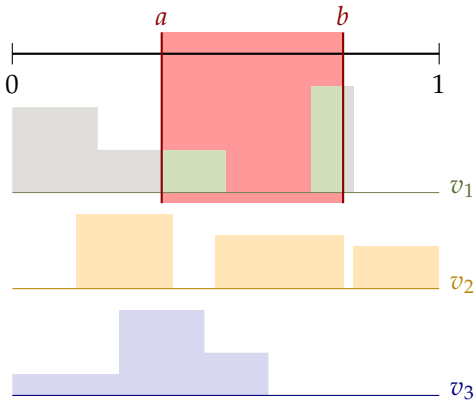

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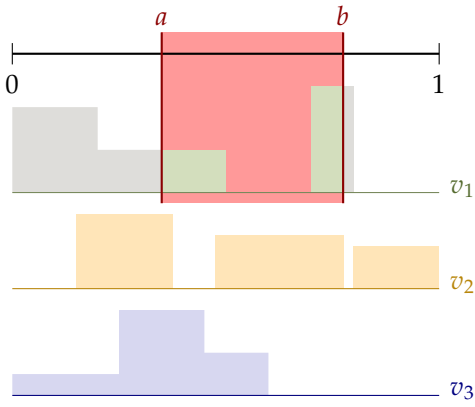

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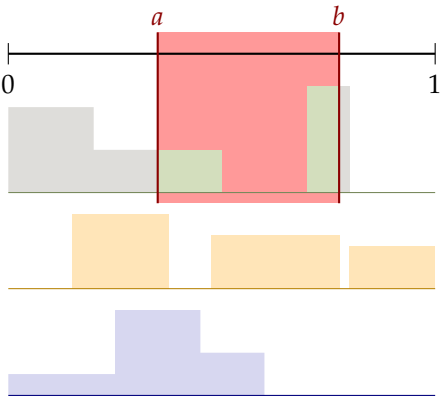

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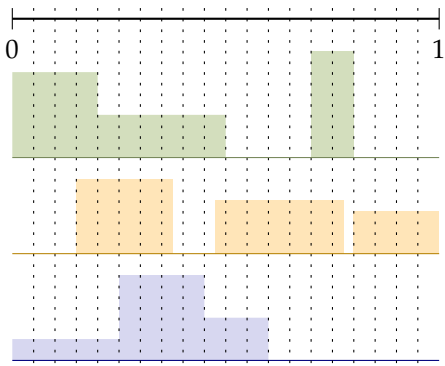

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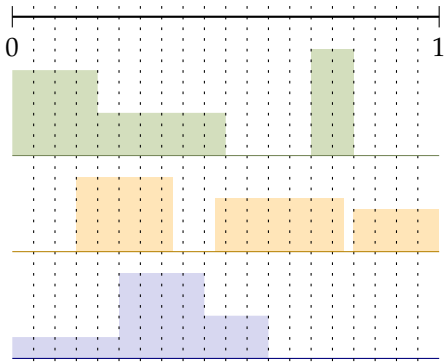

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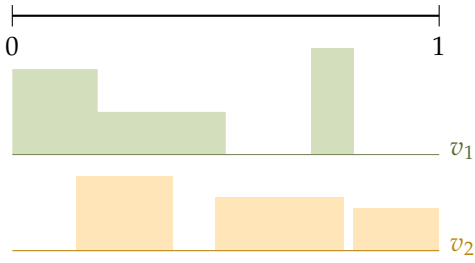
  - Do $\text{polylog}\, 1/\varepsilon$ queries suffice?

Envy-Free Cake Cutting: Two Agents



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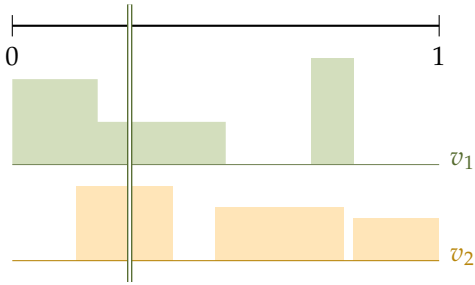

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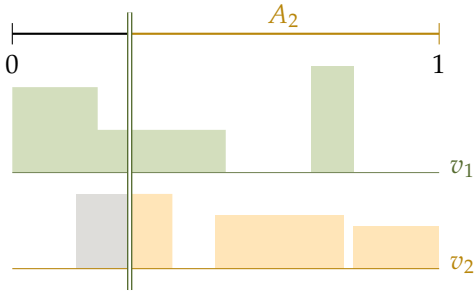

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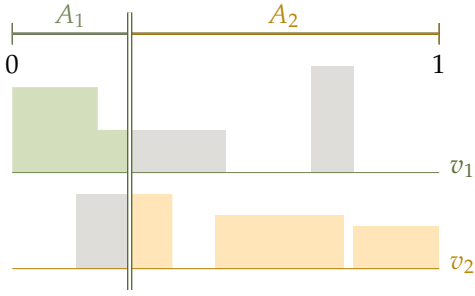

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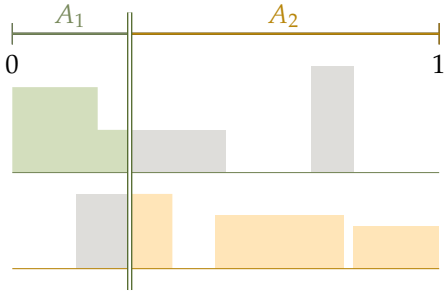

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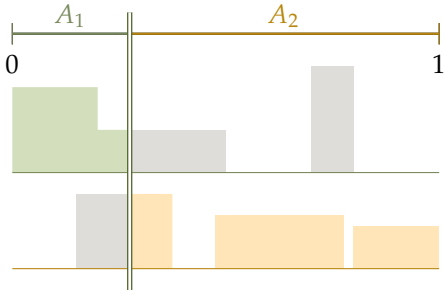

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- Efficient algorithm for two agents
- Works for general valuations

Envy-Free Cake Cutting: Query Complexity

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monotone	$O(\log 1/\varepsilon)$			
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Proof Overview

Theorem Cake cutting with 3 agents is at least as hard as the Jordan curve problem.

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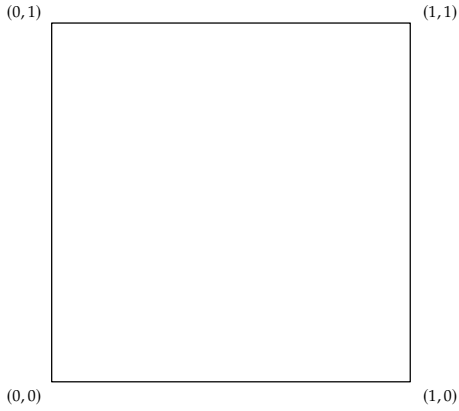
Theorem The Jordan curve problem requires $\Omega(\operatorname{poly} 1/\varepsilon)$ queries.

Jordan Curve



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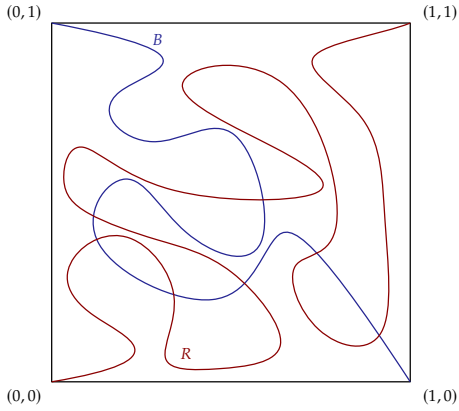

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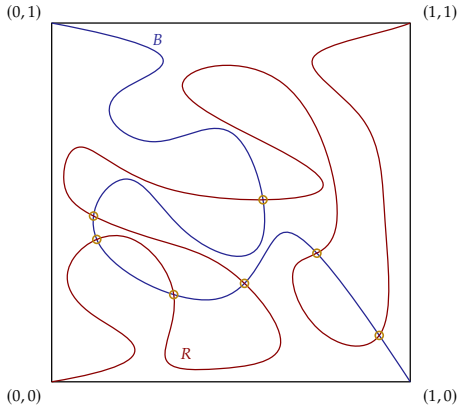

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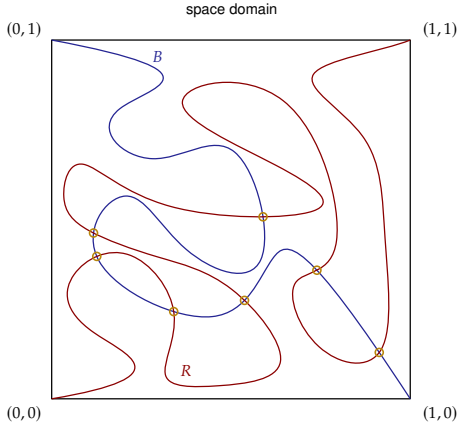

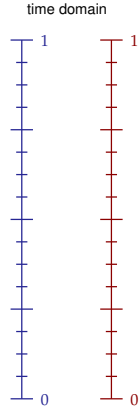

$B, R\colon [0,1] \to [0,1]^2$

Lipschitz-continuous

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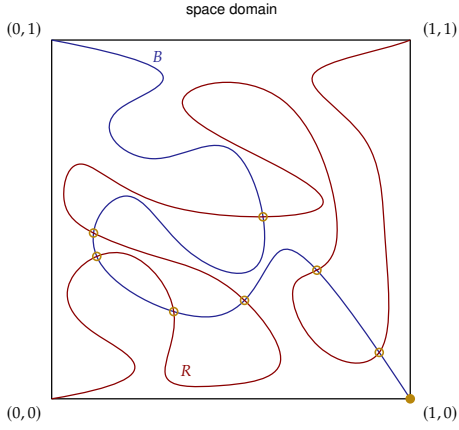

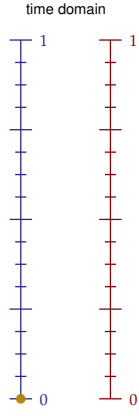

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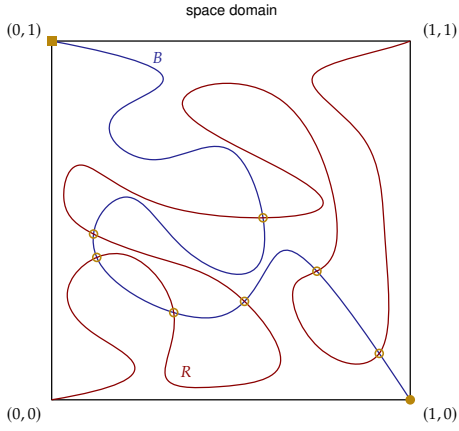

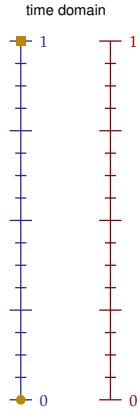

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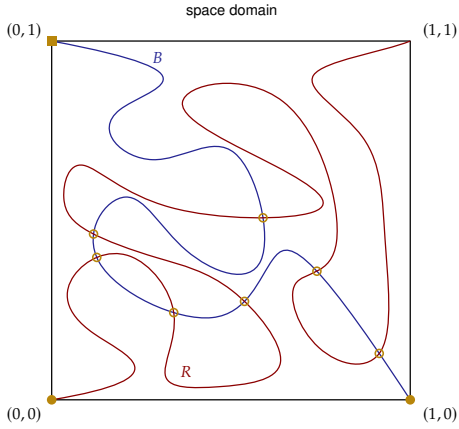

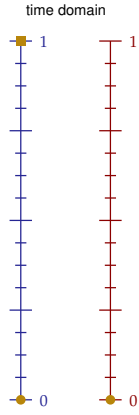

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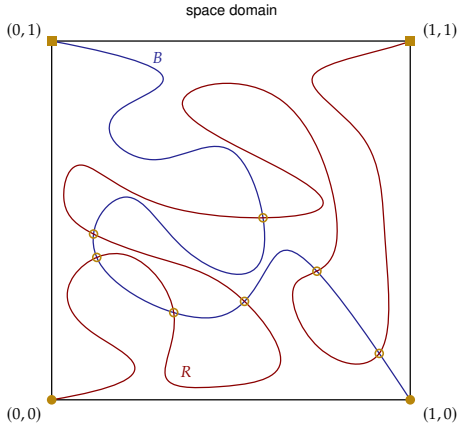

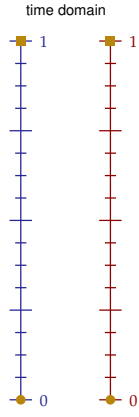

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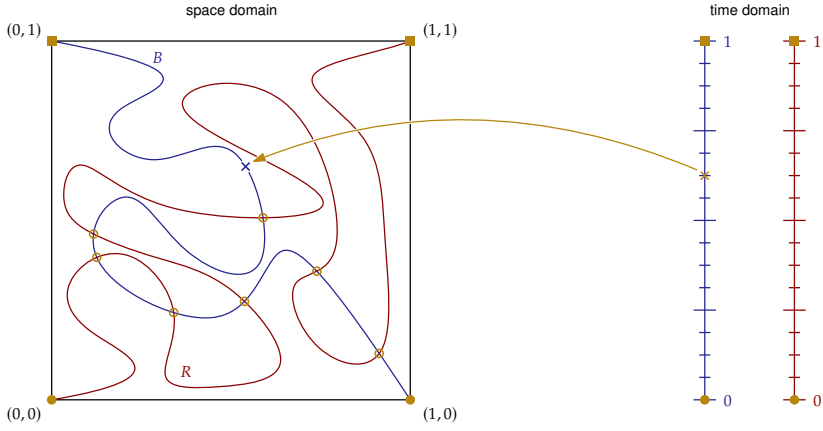
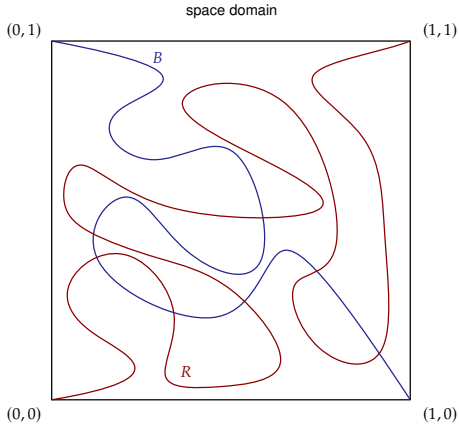
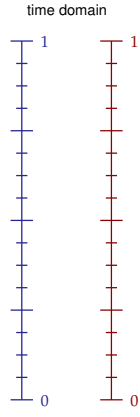


Jordan Curve





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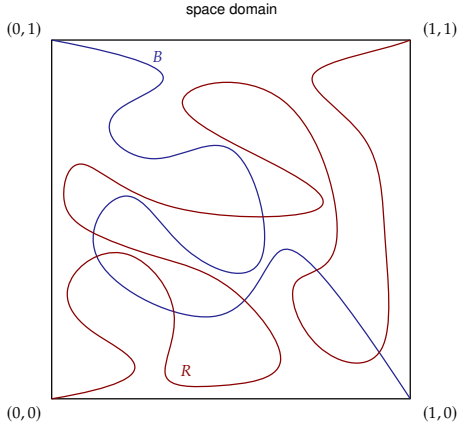

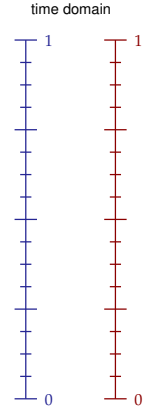

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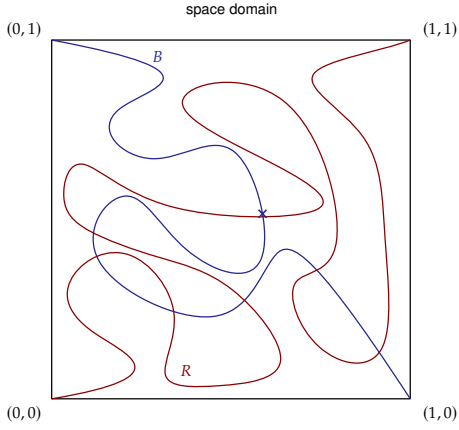

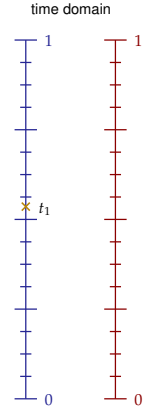

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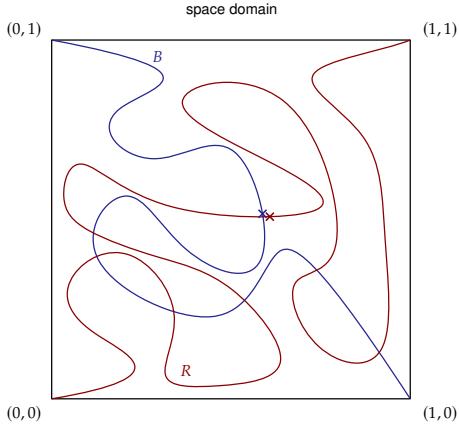

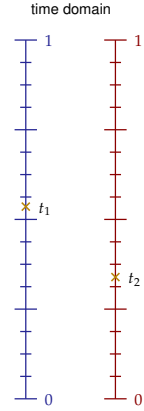

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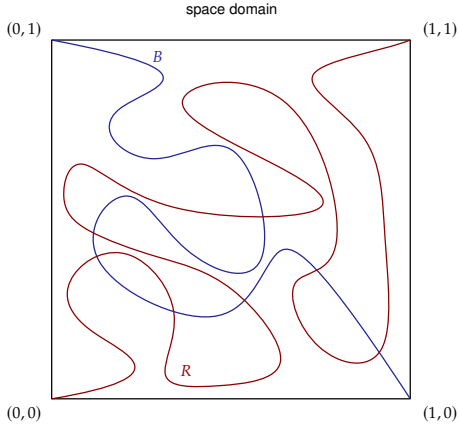

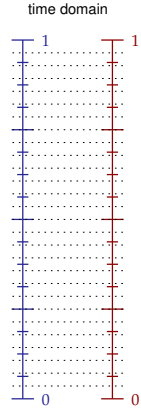

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Theorem Cake cutting with 3-agent is at least as hard as the Jordan curve problem.

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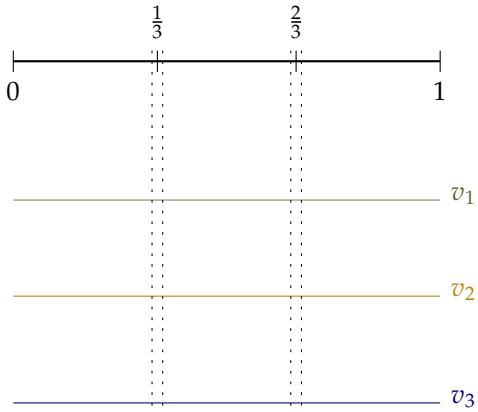

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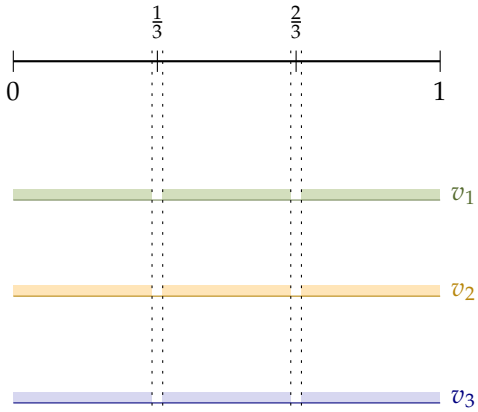

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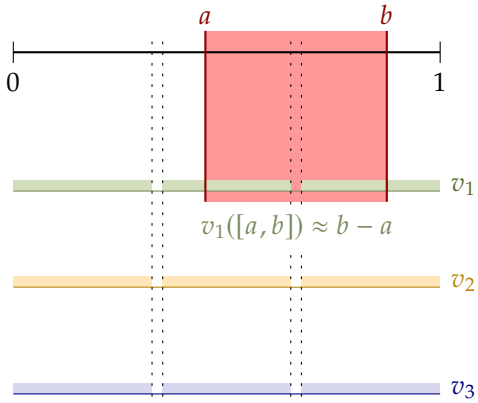

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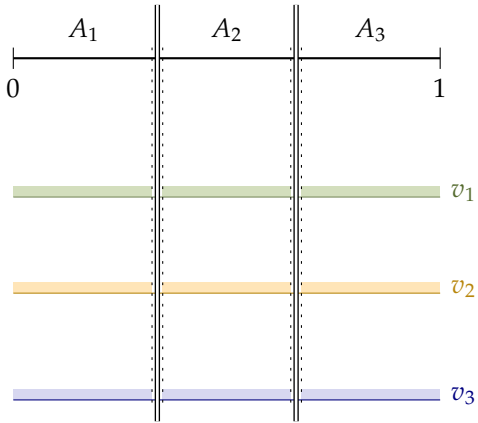

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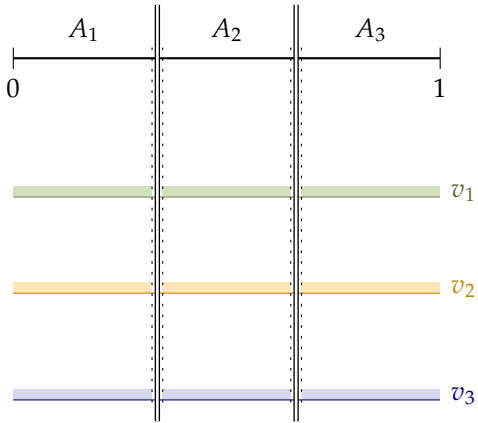

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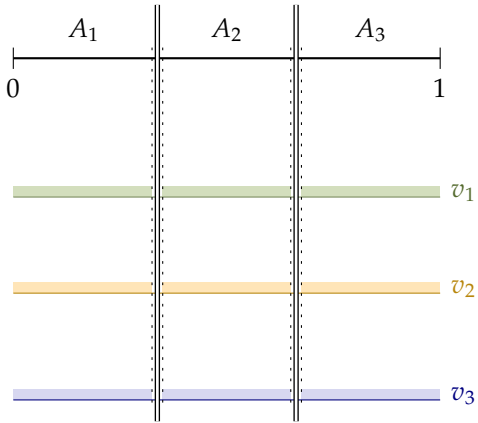

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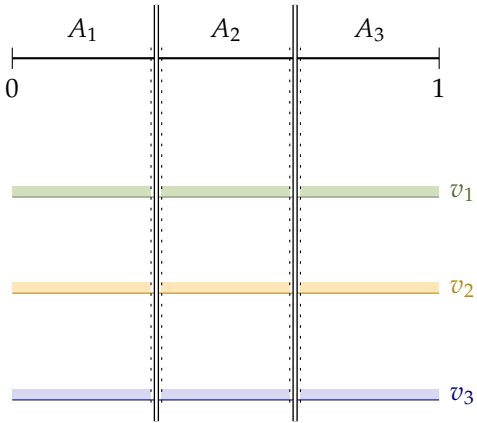

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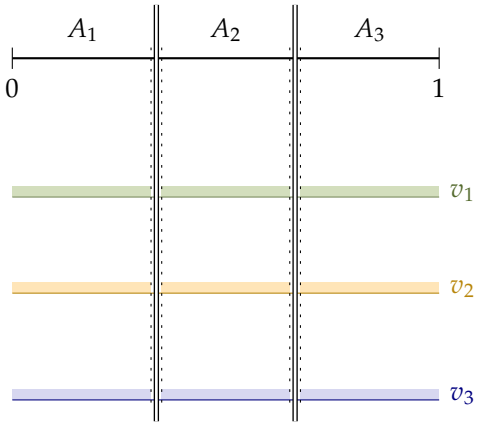

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Proof Overview

Theorem Cake cutting with 3 agents is at least as hard as the Jordan curve problem.

Theorem The Jordan curve problem requires $\Omega(\operatorname{poly} 1/\varepsilon)$ queries.

Proof Overview

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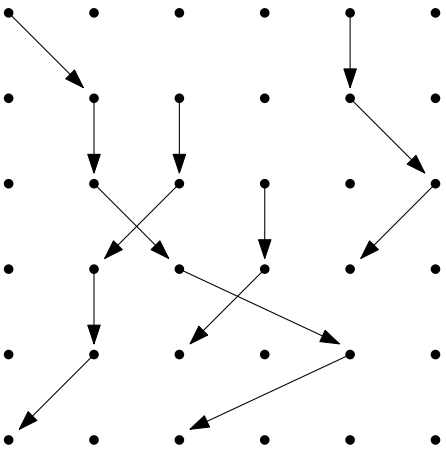
Theorem The Jordan curve problem is at least as hard as the UEoPL problem

Unique End of Potential Line



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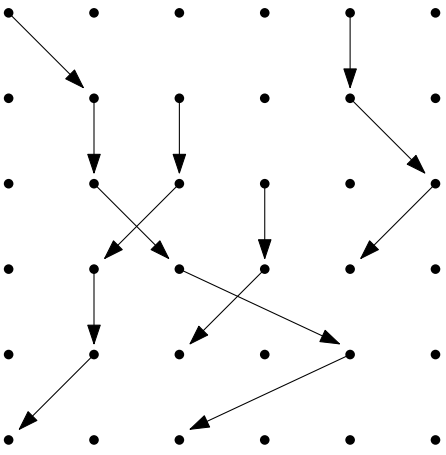

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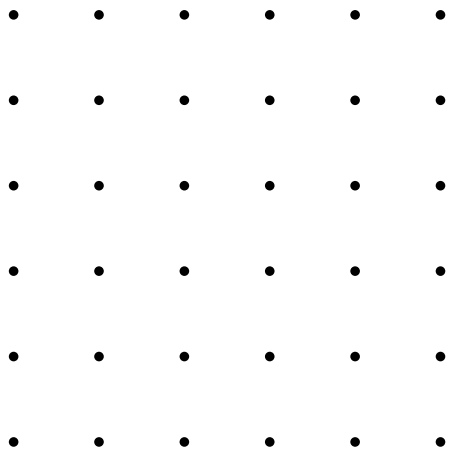

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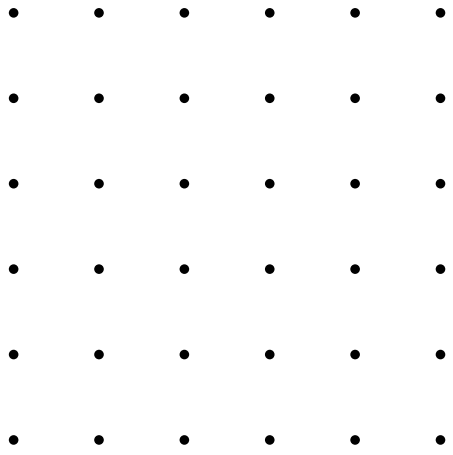

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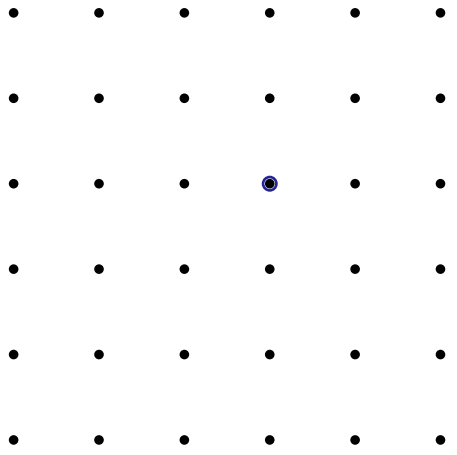

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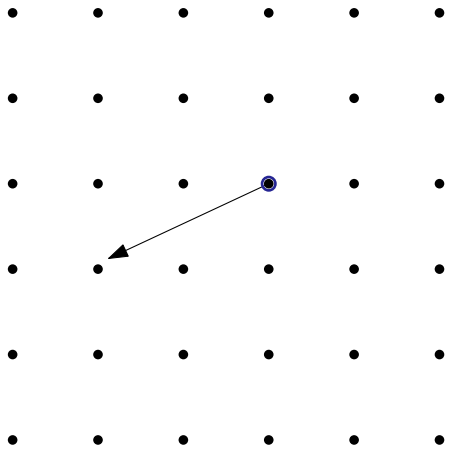

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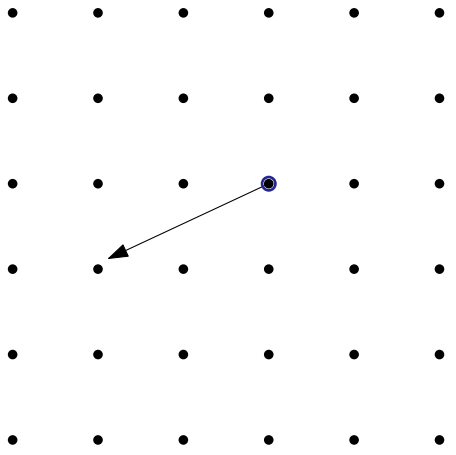

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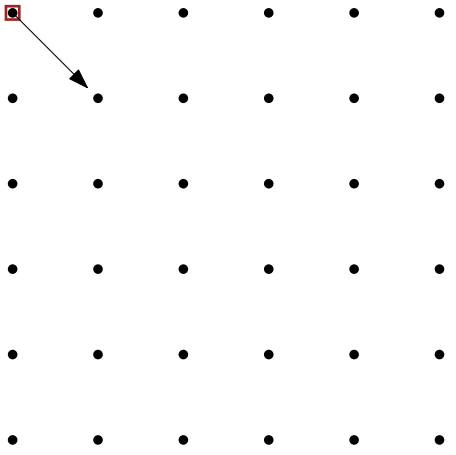

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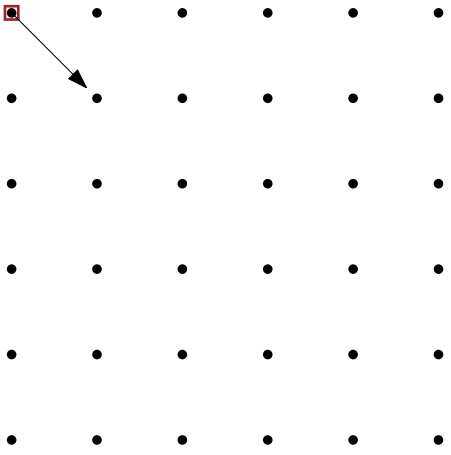

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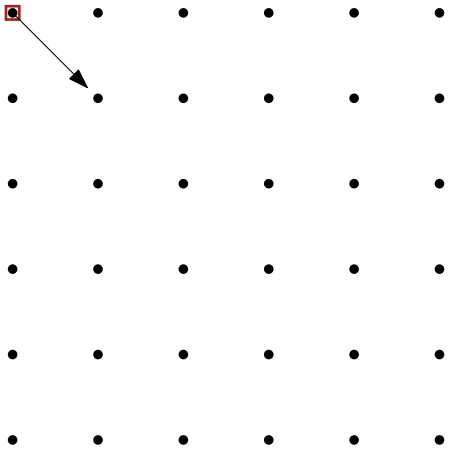

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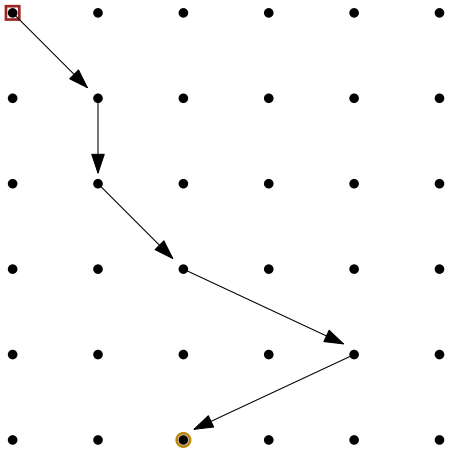

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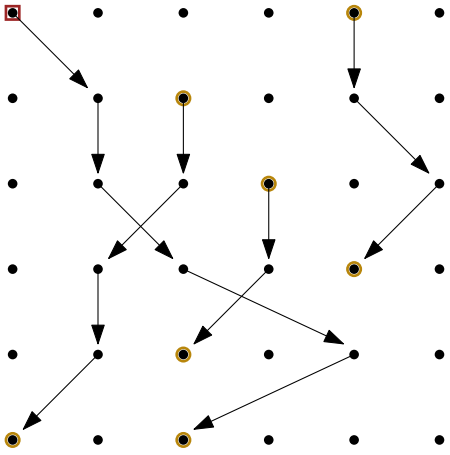

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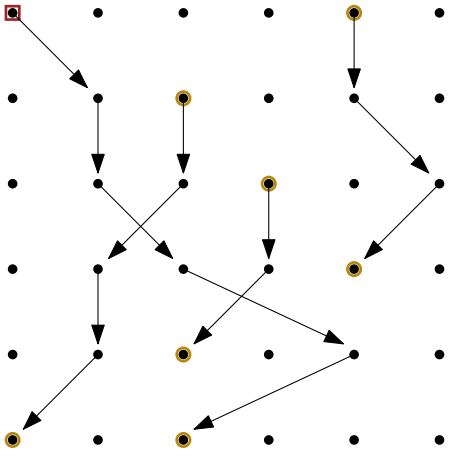

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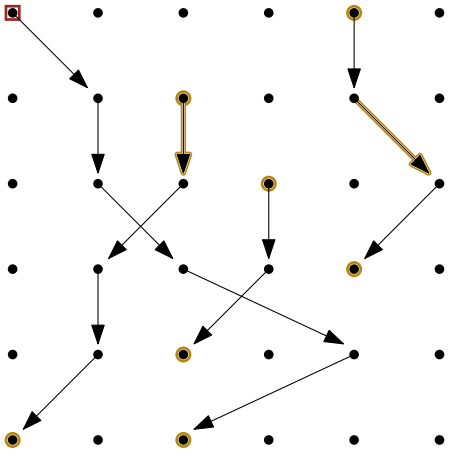

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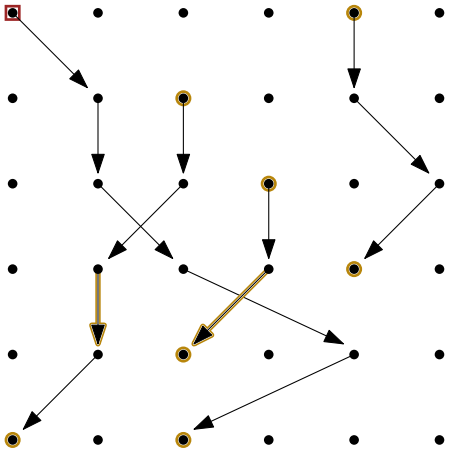

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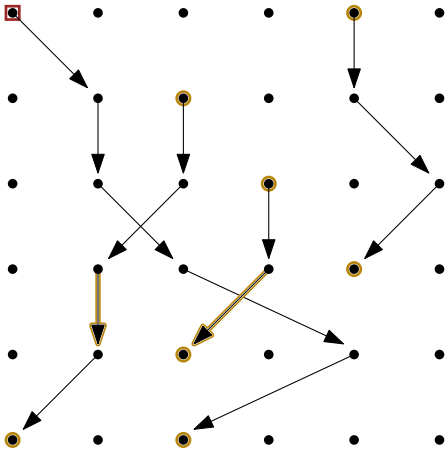

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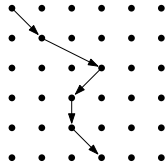
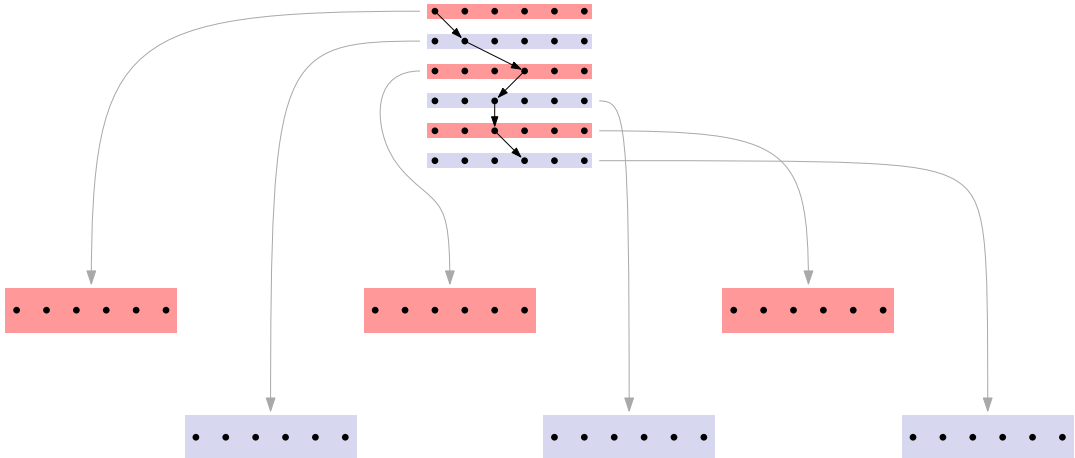
Theorem It requires $\Omega(n^{\varepsilon})$ queries to find a solution.

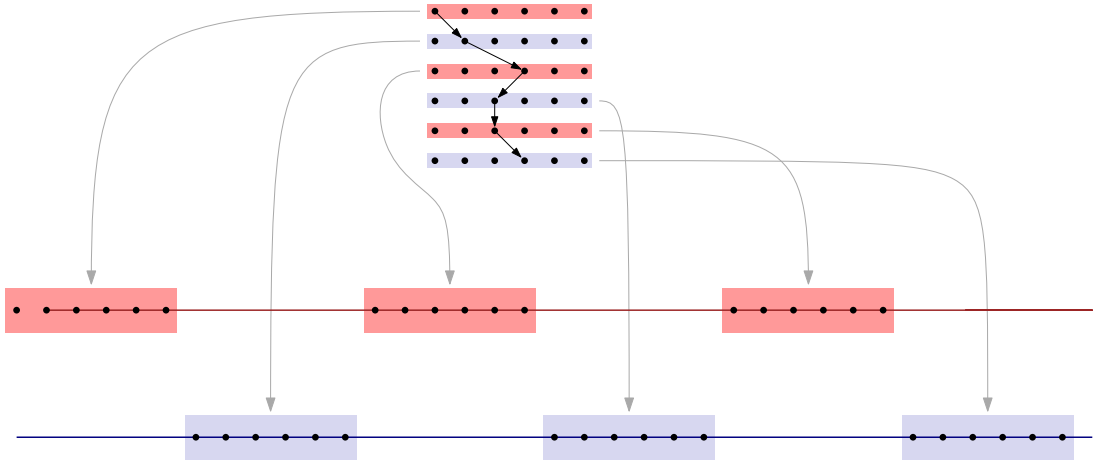
From Unique End of Potential Line to Jordan Curve



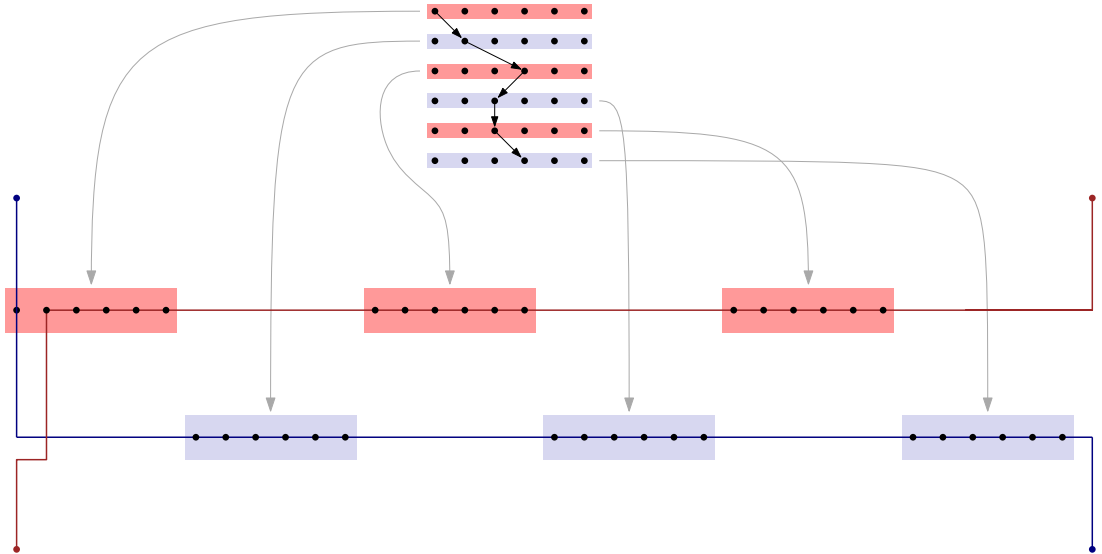
From Unique End of Potential Line to Jordan Curve



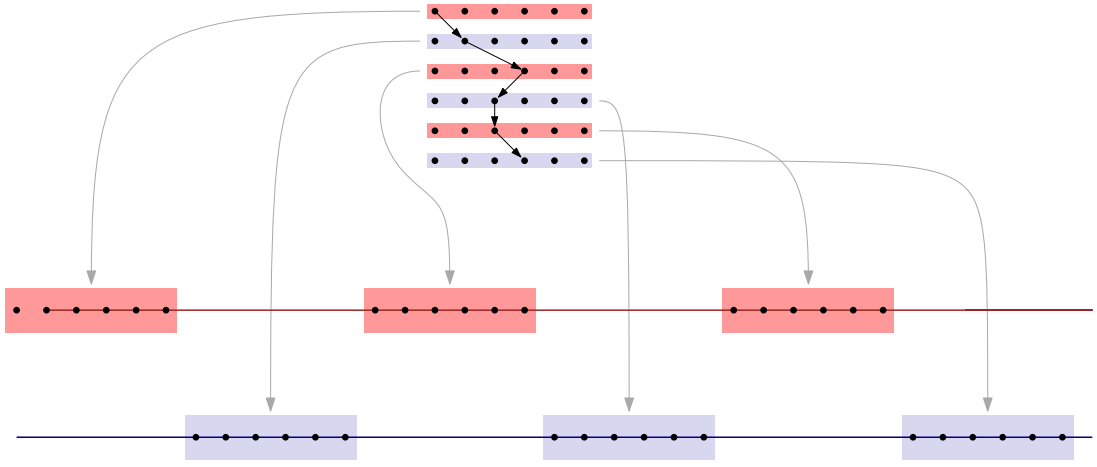
From Unique End of Potential Line to Jordan Curve



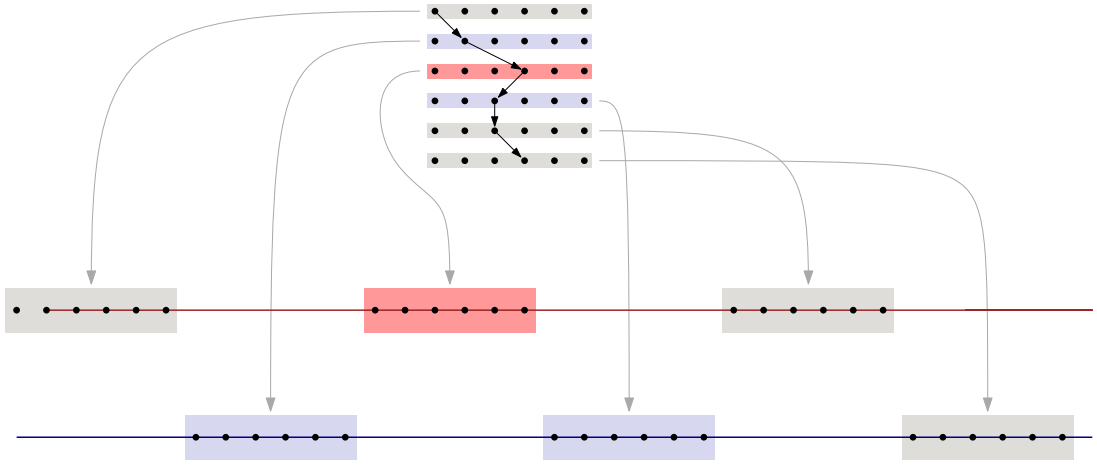
From Unique End of Potential Line to Jordan Curve



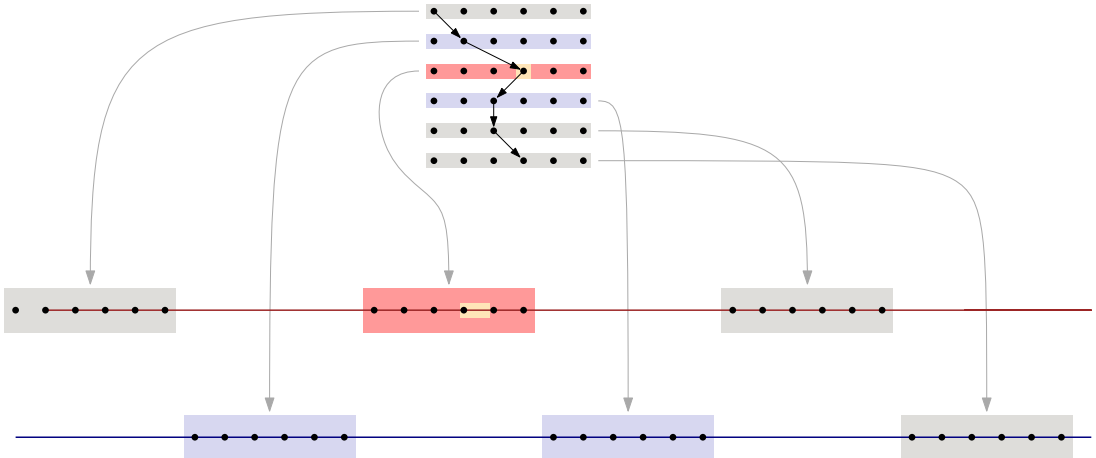
From Unique End of Potential Line to Jordan Curve



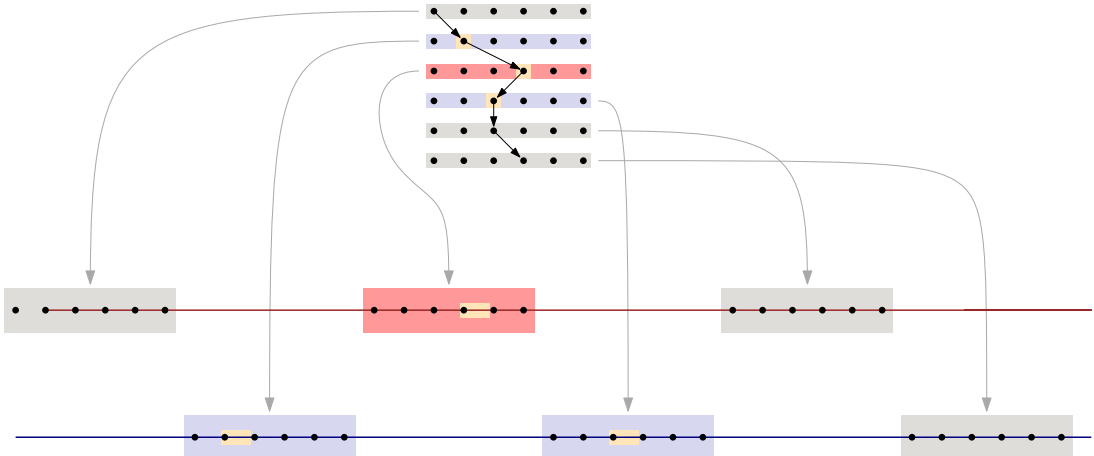
From Unique End of Potential Line to Jordan Curve



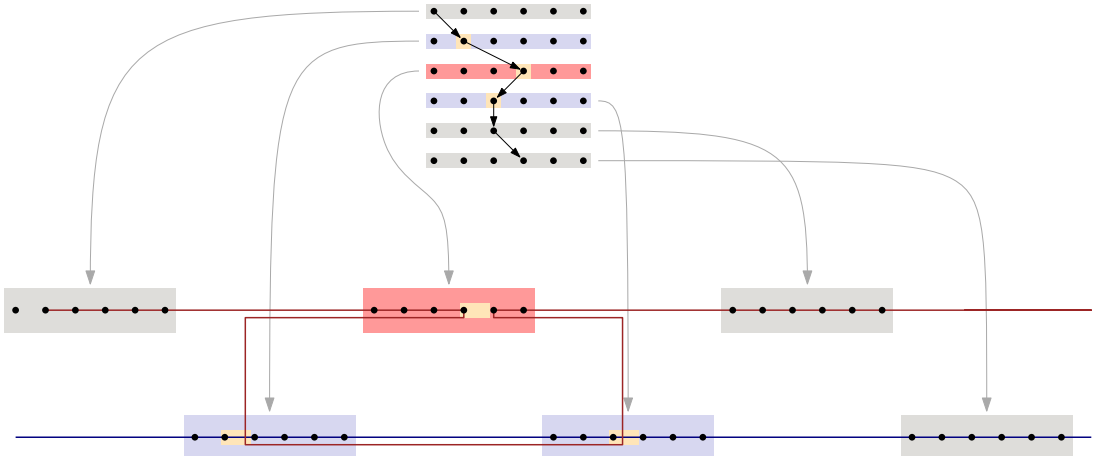
From Unique End of Potential Line to Jordan Curve



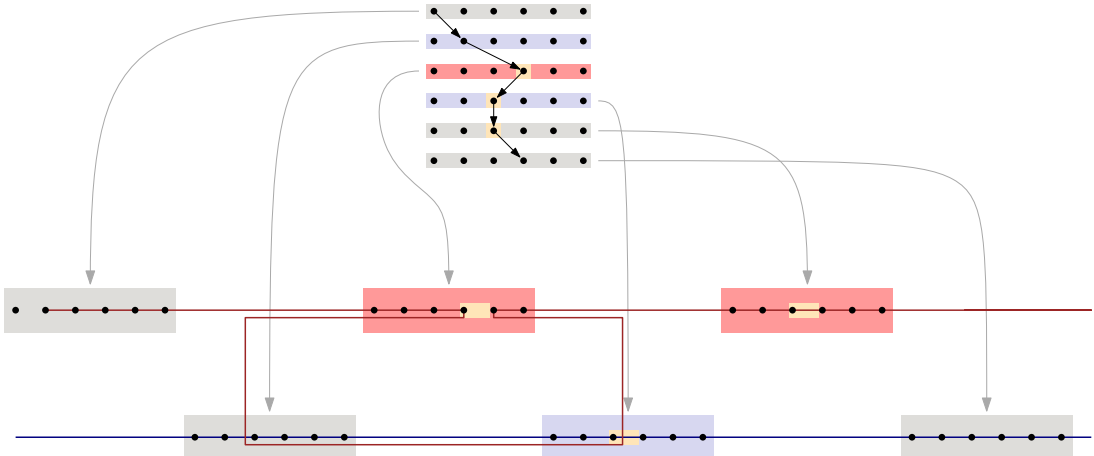
From Unique End of Potential Line to Jordan Curve



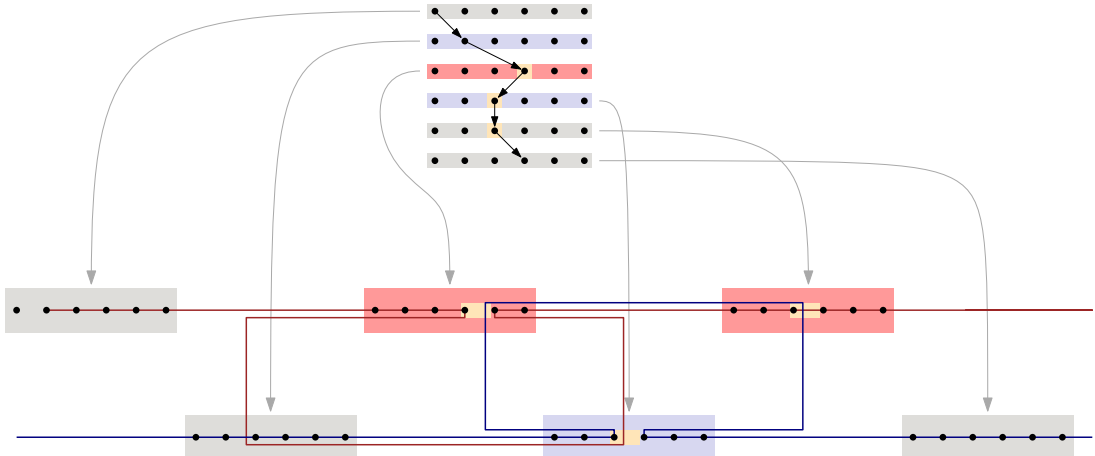
From Unique End of Potential Line to Jordan Curve



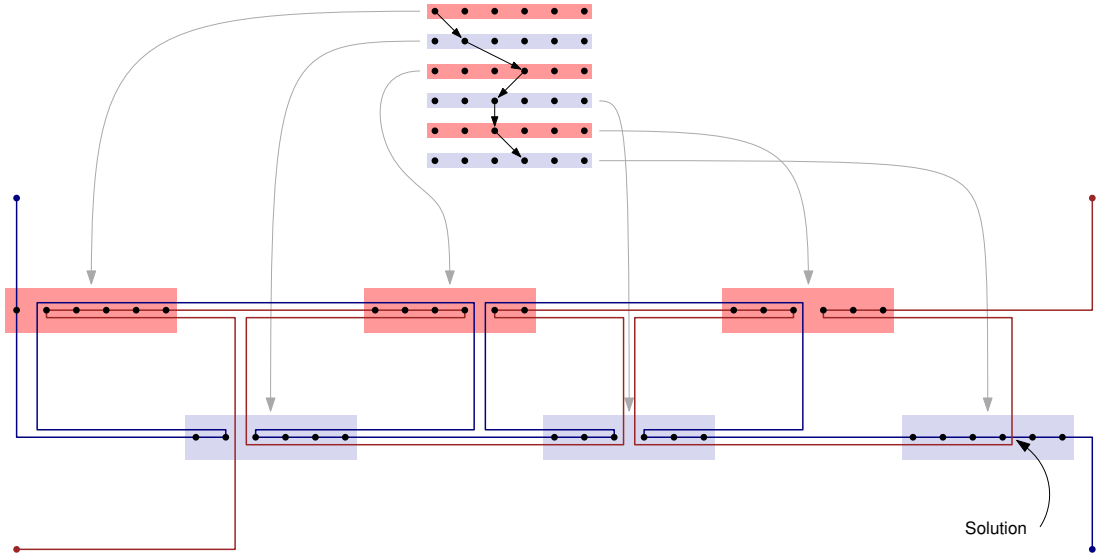
From Unique End of Potential Line to Jordan Curve



From Unique End of Potential Line to Jordan Curve



From Unique End of Potential Line to Jordan Curve



Summary

Theorem Cake cutting with 3 agents and general valuations requires $\Omega(\text{poly}\, 1/\varepsilon)$ queries.

- By reduction: cake cutting ← Jordan curve ← unique end of potential line

Open problems

- Communication complexity of 3 agent cake cutting with general valuations?
- Is there an efficient PLS algorithm for Jordan curve?
- Is 3 agent cake cutting PPAD-complete?
- Query complexity of 5 agent cake cutting with monotone valuations?